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"COMPUTATIONAL METHODS AND IMPLEMENTATION TECHNIQUES FOR PROBLEMS RELATED TO NAVIGATION, GUIDANCE AND CONTROL"

for the period

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## I. PERSONNEL

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## II. TECHNICAL ACCOMPLISHMENTS TO DATE

Our efforts have been evenly devoted to the three objectives of the original proposal. Specifically:

### 1) Computational Methods for Two Point Boundary Value Problems

Recent work[1] has shown that conjugate gradient methods are more efficient than the gradient method for parameter optimization problems with and without side constraints and for optimal programming problems without terminal and in-flight constraints.

An extension of this method is sought to optimal programming problems with terminal constraints, in-flight equality or inequality constraints, and variable time problems.

To gain an insight into the computational aspects, conjugate gradient methods have been applied to a few parameter optimization problems with equality constraints. Work is being done presently to extend this method to optimal programming problems without constraints, with an ultimate extension envisaged to more general problems in trajectory optimization.

### 2) Linear and Nonlinear Filtering

The work during this period has been concerned with two areas. One is the comparison of the statistical analysis of the optimal estimation of the state of a sounding rocket (or similar type of vehicle) during re-entry based on accelerometer measurements with the statistical analysis of the conventional approach to processing the accelerometer data. The second area is the development of a technique for handling the discrete estimation problem when there is colored noise in the measurements leading to an illconditioned problem.[2]

In another application of the linear maximum likelihood filter, a coding scheme was determined for an additive noisy channel with noisy feedback links. This coding scheme allows error-free transmission at a rate only slightly below the Shannon channel capacity and constitutes a significant extension of the recent result of Schalkwijk and Kailath.[3] The key step in the coding scheme is the construction of a filter which estimates the output of another filter.[4]

3) Allocation of Computing Resources

The problem of allocating computing resources in a general time-shared system is formulated as one of stochastic control for finite state systems.[5] By considering choices of queue discipline as control variables, it was shown that under proper assumptions the optimum choice is calculable as a deterministic function of the number of users in the system.

4) Design of Linear Feedback Controllers for Linear Stationary Systems

Using statistical data on anticipated disturbances, controllers can be designed to minimize the expected value of a quadratic form in the state and control variables. We believe that the weighting factors can be determined iteratively to produce acceptable values of mean-square state and control. The technique is being applied to lateral autopilot design for transport-type aircraft with cross-wind disturbances and two controls (aileron and rudder).

5) Explicit Guidance for Orbit Injection

Using the dimensionless state variable approach,[6] the thrust direction for orbit injection has been determined as a function of only two variables for constant gravity and constant thrust acceleration.[7] This should prove useful for orbit injection with small range angle changes and/or in the terminal phases of injection where great precision is required.

## References


1. S. Mitter, L. S. O. Lasdon, and A. D. Warren, "The Method of Conjugate Gradient for Optimal Control Problems," Proc. IEEE, Vol. 54, No. 6, June 1966, p. 904.

2. A. E. Bryson and D. E. Johansen, "Linear Filtering for Time-Varying Systems Using Measurements Containing Colored Noise," IEEE Trans. on Auto. Cont., Vol. AC-10, No. 1, Jan. 1965, pp. 4-10.

3. J. P. M. Schalkwijk and T. Kailath, "A Coding Scheme for Additive Noise Channel with Feedback; Part I: No Bandwidth Constraint," IEEE Trans. on Info. Theory, Vol. IT-12, No. 2, April 1966, pp. 172-182.

4. R. L. Kashyap, "A Sequential Coding Scheme for an Additive Noise Channel with a Noisy Feedback Link," Cruft Lab. Tech. Rept. #508, July 1966.

5. R. L. Kashyap, "Optimization of Stochastic Finite State Systems," Cruft Lab. Tech. Rept. #499, April 1966.

6. A. E. Bryson, "Nonlinear Feedback Solutions for Minimum Time Rendezvous with Constant Thrust Acceleration," 16th International Astronautical Congress, Athens, Greece, September 1965.

7. D. H. Winfield, "Nonlinear Feedback Solution for Minimum-Time Injection into Circular Orbit with Constant Thrust Acceleration Magnitude," Cruft Lab. Tech. Rept. #507, July 1966.


## III. PROPOSED RESEARCH FOR NEXT SIX MONTHS

We intend to continue the research that has been initiated to date.

## IV. REPORTS ISSUED

1. R. L. Kashyap, "A Sequential Coding Scheme for an Additive Noise Channel with a Noisy Feedback Link," Cruft Lab. Tech. Rept. #508, July 1966.

2. R. L. Kashyap, "Optimization of Stochastic Finite State Systems," Cruft Lab. Tech. Rept. #499, April 1966.

3. D. H. Winfield, "Nonlinear Feedback Solution for Minimum-Time Injection into Circular Orbit with Constant Thrust Acceleration Magnitude," Cruft Lab. Tech. Rept. #507, July 1966.

## V. PAPERS PUBLISHED

1. R. L. Kashyap and C. C. Blaydon, "Recovery of Functions from Noisy Measurements Taken at Randomly Selected Points and its Application of Pattern Classification," Proc. IEEE, Vol. 54, No. 8, Aug. 1966, pp. 1127-1129.